P-FUZZY IDEALS AND H-FUZZY IDEALS IN BCI ALGEBRAS

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ABSTRACT

In this paper we investigate P-fuzzy ideals and H-fuzzy ideals in BCI-algebras. After recalling the notions of ideals, P-ideals and H-ideals, we consider fuzzy subsets satisfying suitable P-type and H-type conditions and regard them as P-fuzzy and H-fuzzy ideals, respectively. For a P-fuzzy (resp. H-fuzzy) ideal f of a BCI-algebra X, we study the level subset A_f and prove that A_f is a crisp P-ideal (resp. H-ideal) of X. This establishes a direct connection between fuzzy ideals and their corresponding classical counterparts. Furthermore, by using a simple lemma on infima of families of real numbers, we show that arbitrary pointwise infima of P-fuzzy ideals (resp. H-fuzzy ideals) are again P-fuzzy ideals (resp. H-fuzzy ideals). Several examples on a finite BCI-algebra are provided to illustrate the main results. These structural properties supply basic tools for the later study of the lattice-theoretic behavior of P-fuzzy and H-fuzzy ideals in BCI-algebras.

Keywords: BCK/BCI Algebra, ideal, P-ideal, H-ideal, P-fuzzy ideal, H-fuzzy ideal.

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P− مثالية ضبابية و H− مثالية ضبابية في جبور BCI

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الملخص:

في هذا البحث ندرس المثاليّات الضبابية من النوع P والمثاليّات الضبابية من النوع H في جبر BCI بعد استعراض مفاهيم المثالية ، وP− مثالية و H− مثالية ، نفرض المجموعات الضبابية التي تحقق شروطاً مناسبة من النوع P وH، نسميها على الترتيب مثاليّات ضبابية من النوع P ومثاليّات ضبابية من النوع H ولكل مثالية ضبابي من النوع P (وعلى الترتيب من النوع H) معرَّف بدالة ضبابية f على جبر BCI يرمز له بـ X ، ندرس مجموعة المستوى A_f ونبرهن أن A_fمثالي من النوع P (وعلى الترتيب من النوع H) في X وبذلك نحصل على ارتباط مباشر بين المثاليّات الضبابية ونظيراتها الكلاسيكية.

علاوةً على ذلك، وباستخدام مبرهنة مساعدة بسيطة تتعلّق بـ inf لأسرة من الأعداد الحقيقية، نبيّن أن inf للمثاليّات الضبابية من النوع P (وعلى الترتيب من النوع H) هي بدورها مثاليّات ضبابية من النوع P (وعلى الترتيب من النوع H) كما نقدّم عدّة أمثلة على جبر BCI منتهٍ لتوضيح النتائج الرئيسة. وتوفّر هذه الخصائص البنيوية أدواتٍ أساسيةً للدراسة اللاحقة للسلوك الشبكي للمثاليّات الضبابية من النوعين P و H في جبر BCI.

الكلمات المفتاحية: جبر BCK/BCI, مثالية، P− مثالية، H− مثالية، P− مثالية ضبابية، H− مثالية ضبابية.

1. Introduction

The study of BCK/BCI algebra was initiated by Imai and Iseki [13] in 1966 as a generalization of the concept of set-theoretic difference and propositional calculi. The concept of fuzzy sets was introduced by L. A. Zadeh [8]. And Rosenfeld [1] the first to apply the concept of fuzzy sets to algebraic systems in 1971. And O.Xi applied the concept of fuzzy sets on BCK-algebras. With the development of interest in fuzzy algebraic structures, the study of fuzzy ideals and their various forms in BCK-algebras have gained considerable importance [10], due to the role they play in characterizing the properties of these algebras and organizing their internal structure. This led to the emergence of several notions of fuzzy ideals, depending on the algebraic framework and the nature of the relationships within it. Among these notions are fuzzy dual ideals and anti-fuzzy dual ideals [7-11], both of which have attracted increasing attention in recent literature.

Recently, a few authors have extended the study of fuzzy ideals in BCK/BCI-algebras to more general frameworks. For instance, neuromorphic and multi-valued versions of ideals have been investigated to model richer uncertainty structures [14]. Other works have focused on bipolar and interval-valued fuzzy ideals [15], which provide more flexible tools for capturing positive and negative information in logical algebraic systems. These contributions confirm that fuzzy ideals and their generalizations in BCK/BCI-algebras are still an active research topic and motivate the investigation carried out in the present paper.

2. Preliminaries

Definition 2.1. [4-6] Let $(X,*,0)$ be a groupoid with a distinguished element 0 and a binary operation $*$. Then $(X,*,0)$ is a BCI−algebra if:

(I) $(\forall x,y,z \in X)\left(\left((x*y)*(x*z)\right)*(z*y)=0\right)$,
(II) $(\forall x,y \in X)\left(\left(x*(x*y)\right)*y=0\right)$,
(III) $(\forall x \in X)$ $(x*x=0)$,
(IV) $(\forall x,y \in X)$ $(x*y=0 \text{ and } y*x=0 \text{ imply } x=y)$.

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In a BCI−algebra X, a partially ordered relation $\leq$ can be delned by

$(\forall x, y \in X)(x \leq y \Leftrightarrow x * y = 0)$.

A BCI algebra is said to be a BCK algebra if it satisfies:

(V) $(\forall x \in X)$ $(0 * x = 0)$

A BCK/BCI−algebra X has the following properties: [12]

(b1) $(\forall x \in X)$ $(x * 0 = x)$.

(b2) $(\forall x, y, z \in X)$ $((x * y) * z = (x * z) * y)$.

(b3) $(\forall x, y, z \in X)$ $(x \leq y \Rightarrow x * z \leq y * z,\ z * y \leq z * x)$.

(b4) $(\forall x, y, z \in X)$ $((x * z) * (y * z) \leq x * y)$.

If X is a BCK−algebra, then the following property hold:

(b5) $(\forall x, y \in X)$ $((x * y) * x = 0)$.

Definition 2.2. [9] A BCK−algebra X is said to be bound if there exists an element $1 \in X$ such that $(\forall x \in X)$ $(x \leq 1)$.

For elements x and y of a bound BCK−algebra X,

we denote [2] $x \vee y = N(Nx \wedge Ny)\ where\ Nx = 1 * \mathrm{x}$

and $a \vee b = N(Na \wedge Nb)$; $\forall\, a, b \in A$

Definition 2.3. [5] A BCK−algebra X is called commutative if

$(\forall x, y \in X)$ $(\, x * (x * y) = y * (y * x))$ where $x \wedge y = y * (y * x)$

Definition 2.4. [6] A nonempty subset I of a BCK/BCI−algebra X is said to be an ideal if it satisfies:

(I1) $0 \in I$

(I2) $(\forall x, y \in X)\ (x * y \in I \ and\ y \in I\ imply\ x \in I)$

Any ideal I have the property: (I3) $x \in I$ and $y \leq x$ imply $y \in I$

Definition 2.5. [3] Let $I \subseteq X$ be a non−empty subset.

(a) I is a P−ideal if:

(P1) $0 \in I$.

(P2) $(\forall x, y, z \in X)\ (if (x * z) * (y * z) \in I\ and\ y \in I, then\ x \in I)$.

(b) I is an H−ideal if:

(H1) $0 \in I$.

(H2) $(\forall x, y, z \in X)\ (if\ x * (y * z) \in I\ and\ y \in I, then\ x * z \in I)$.

Definition 2.6. [3] Let X be a set. A fuzzy set f in X is a function $f: X \rightarrow [0, 1]$. And the complement of f, denoted by f^c is the fuzzy subset of X given by $f^c(x) = 1 - f(x)\ \ for\ all\ x \in X$

Definition 2.7. [3] A fuzzy set f in a BCI−algebra X is said to be a fuzzy ideal of X if it satisfies:

(F1) $(\forall x \in X)\ (f(0) \geq f(x))$,

(F2) $(\forall x, y \in X)\ (f(x) \geq\ min\{f(x * y), f(y)\})$.

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If f is fuzzy ideal in X then, $(\forall x, y, z \in X)$, such that $(x * y) * z = 0$, then $f(x) \geq min\{f(y), f(z)\}$

Definition 2.8. [3] A fuzzy subset $f : X \to [0,1]$ is called a P−fuzzy ideal if:

$(\mathrm{P-F1})$ $f(0) \geq f(x)$; $\forall x \in X$

$(\mathrm{P-F2})$ $f(x) \geq min\Big(f\big((x * z) * (y * z)\big), f(y)\Big); \forall x, y, z \in X$

Definition 2.9. [3] A fuzzy subset $f : X \to [0,1]$ is called an H−fuzzy ideal if:

$(\mathrm{H-F1})$ $f(0) \geq f(x)$; $\forall x \in X$

$(\mathrm{H-F2})$ $f(x * z) \geq f\big(x * (y * z)\big) \wedge f(y); \forall x, y, z \in X$

3. Main results

In this section we establish some basic structural properties of P−fuzzy ideals and H−fuzzy ideals in BCI−algebras. We show that the level set at the value $f(0)$ of such fuzzy ideals are crisp P−ideals and H−ideals, respectively, and that arbitrary pointwise infima of P−fuzzy (resp. H−fuzzy) ideals remain inside the same class. These facts will be used later to describe the lattice structure.

Theorem 3.1. let X BCI Algebra and let $f : X \to [0,1]$ be a fuzzy subset of X. Define $A_f = \{ x \in X ; f(x) = f(0) \}$.

If f is a P-fuzzy ideal of X, then A_f is a P-ideal of X.

Proof. Since $f(0) = f(0)$, we have $0 \in A_f$, so (P1) holds.

To show (P2): assume $(x * z) * (y * z) \in A_f$ and $y \in A_f$. Then $f((x * z) * (y * z)) = f(0)$ and $f(y) = f(0)$.

From (P−F2): $f(x) \geq min(f((x * z) * (y * z)), f(y))$

$$= min(f(0), f(0)) = f(0).$$

From (P−F1): $f(0) \geq f(x)$.

So $f(0) \geq f(x) \geq f(0) \Rightarrow f(x) = f(0) \Rightarrow x \in A_f$. Hence A_f is a P−ideal.

Theorem 3.2. let X BCI Algebra and let $f : X \to [0,1]$ be a fuzzy subset of X. Define $A_f = \{x \in X ; f(x) = f(0)\}$.

If f is a H-fuzzy ideal of X, then A_f is a H-ideal of X.

Proof. The first condition follows immediately from the previous theorem. so $0 \in A_f$.

For (H2): assume $x * (y * z) \in A_f$ and $y \in A_f$. Then

$f(x * (y * z)) = f(0), f(y) = f(0)$.

From (H−F2): $f(x * z) \geq min(f(x * (y * z)), f(y))$

$$= min(f(0), f(0)) = f(0).$$

From (H−F1): $f(0) \geq f(x * z)$.

So $f(0) \geq f(x * z) \geq f(0)$, hence $f(x * z) = f(0), i.e. x * z \in A_f$. Therefore A_f is an H−ideal.

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Example 3.3. Let $X = \{0, 1, 2\}$ and define a binary operation $*$ on X by

*	0	1	2
0	0	0	0
1	1	0	0
2	2	2	0

It is well known that $(X,*,0)$ is a BCI–algebra.

Define a fuzzy subset $f_p : X \to [0,1]$ by

$f_p(0) = f_p(1) = f_p(2) = 0.6$

That is, f_p is the constant fuzzy set with value 0.6 on all elements of X.

f_p is a P–fuzzy ideal, for f_p we have $f_p(0) = 0.6$ and, for every x in X,

$f_p(x) = 0.6$. Hence $f_p(0) = 0.6 \geq 0.6 = f_p(x)$ for all x in X.

So $(\mathrm{P - F1})$ holds.

Take arbitrary x, y, z in X. Since f_p is constant,

$f_p(x) = f_p((\mathrm{x} * \mathrm{z}) * (\mathrm{y} * \mathrm{z})) = f_p(y) = 0.6$

Therefore, $min\{ f_p((\mathrm{x} * \mathrm{z}) * (\mathrm{y} * \mathrm{z})), f_p(y)\} = \min\{0.6,0.6\} = 0.6.$

Thus $f_p(x) = 0.6 \geq 0.6 = \min\{f_p((\mathrm{x} * \mathrm{z}) * (\mathrm{y} * \mathrm{z})), f_p(y)\}$ for all x, y, z in X. So, condition $(\mathrm{P - F2})$ also holds. Consequently, f_p is a P–fuzzy ideal of X.

And $A_f = \{ x \in X ;\ f(x) = f(0)\} = \{0,1,2\} = X$. Which is trivially a P−ideal (the whole algebra)

Example 3.4. Again $X = \{0,1,2\}$ with the same BCI−structure.

Define $f_H : X \to [0,1]$ by $f_H(0) = 0.7, f_H(1) = 0.6, f_H(2) = 0.4$

Clearly: $0.7 \geq 0.6$ and $0.7 \geq 0.4$, so $f_H(0) \geq f_H(1)$ and $f_H(0) \geq f_H(2)$.Therefore $(\mathrm{H}-\mathrm{F1})$ holds. Now we must check the following inequality for all $x, y, z\ in\ X$:

$(\mathrm{H}-\mathrm{F2})\ \ f_H(x * z) \geq f_H\big(x * (y * z)\big) \wedge f_H(y)\ \ ;\ \forall x, y, z \in X$

Below we verify this explicitly for all 27 triples (x, y, z)

For $z = 0$, we compute: $x * z, y * z, x * (y * z), f_H(x * z), f_H\big(x * (y * z)\big), f_H(y)$.

$RHS = min\{ f_H(x * (y * z)), f_H(y) \}$, and compare $f_H(x * z) \geq RHS$.

From the table: $0 * 0 = 0,\ 1 * 0 = 1,\ 2 * 0 = 2$.

$z = 0$, full list of all (x, y)

$(1)\ x = 0, y = 0: x * z = 0 * 0 = 0, y * z = 0 * 0 = 0,\ x * (y * z) = 0 * 0 = 0$

$f_H(x * z) = f_H(0) = 0.7$, $f_H(x * (y * z)) = f_H(0) = 0.7$, $f_H(y) = f_H(0) = 0.7$

$RHS = min(0.7, 0.7) = 0.7$. So, $0.7 \geq 0.7$ (true)

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(2) $x = 0, y = 1$: $x * z = 0 * 0 = 0, y * z = 1 * 0 = 1, \ x * (y * z) = 0 * 1 = 0$

$f_H(x * z) = f_H(0) = 0.7$, $f_H(x * (y * z)) = f_H(0) = 0.7$, $f_H(y) = f_H(1) = 0.6$

$RHS = min(0.7, 0.6) = 0.6$. So, $0.7 \geq 0.6$ (true)

(3) $x = 0, y = 2$: $x * z = 0 * 0 = 0, y * z = 2 * 0 = 2, \ x * (y * z) = 0 * 2 = 0$

$f_H(x * z) = f_H(0) = 0.7$, $f_H(x * (y * z)) = f_H(0) = 0.7$, $f_H(y) = f_H(2) = 0.4$

$RHS = min(0.7, 0.4) = 0.4$. So, $0.7 \geq 0.4$ (true)

(4) $x = 1, y = 0$: $x * z = 1 * 0 = 1, y * z = 0 * 0 = 0, \ x * (y * z) = 1 * 0 = 1$

$f_H(x * z) = f_H(1) = 0.6$, $f_H(x * (y * z)) = f_H(1) = 0.6$, $f_H(y) = f_H(0) = 0.7$

$RHS = min(0.6, 0.7) = 0.6$. So, $0.6 \geq 0.6$ (true)

(5) $x = 1, y = 1$: $x * z = 1 * 0 = 1, y * z = 1 * 0 = 1, \ x * (y * z) = 1 * 1 = 0$

$f_H(x * z) = f_H(1) = 0.6$, $f_H(x * (y * z)) = f_H(0) = 0.7$, $f_H(y) = f_H(1) = 0.6$

$RHS = min(0.7, 0.6) = 0.6$. So, $0.6 \geq 0.6$ (true)

(6) $x = 1, y = 2$: $x * z = 1 * 0 = 1, y * z = 2 * 0 = 2,\ x * (y * z) = 1 * 2 = 0$

$f_H(x * z) = f_H(1) = 0.6$, $f_H(x * (y * z)) = f_H(0) = 0.7$, $f_H(y) = f_H(2) = 0.4$

$RHS = min(0.7, 0.4) = 0.4$. So, $0.6 \geq 0.4$ (true)

(7) $x = 2, y = 0$: $x * z = 2 * 0 = 2, y * z = 0 * 0 = 0,\ x * (y * z) = 2 * 0 = 2$

$f_H(x * z) = f_H(2) = 0.4$, $f_H(x * (y * z)) = f_H(2) = 0.4$, $f_H(y) = f_H(0) = 0.7$

$RHS = min(0.4, 0.7) = 0.4$. So, $0.4 \geq 0.4$ (true)

(8) $x = 2, y = 1$: $x * z = 2 * 0 = 2, y * z = 1 * 0 = 1,\ x * (y * z) = 2 * 1 = 2$

$f_H(x * z) = f_H(2) = 0.4$, $f_H(x * (y * z)) = f_H(2) = 0.4$, $f_H(y) = f_H(1) = 0.6$

$RHS = min(0.4, 0.6) = 0.4$. So, $0.4 \geq 0.4$ (true)

(9) $x = 2, y = 2$: $x * z = 2 * 0 = 2, y * z = 2 * 0 = 2,\ x * (y * z) = 2 * 2 = 0$

$f_H(x * z) = f_H(2) = 0.4$, $f_H(x * (y * z)) = f_H(0) = 0.7$, $f_H(y) = f_H(2) = 0.4$

$RHS = min(0.7, 0.4) = 0.4$. So, $0.4 \geq 0.4$ (true)

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So $(\mathrm{H}-\mathrm{F2})$ holds for all pairs (x,y) when $z=0$.

Similarly, Case $z=1$ and $z=2$.

We have checked ALL 27 triples $(x,y,z)\in\{0,1,2\}$ and in each

case we obtained: $f_H(x*z)\geq f_H\big(x*(y*z)\big)\wedge f_H(y)$. Therefore, the fuzzy set f_H given by $f_H(0)=0.7, f_H(1)=0.6, f_H(2)=0.4$ is an H–fuzzy ideal on X.

Then $A_{f_H}=\{\,x\mid f_H(x)=f_H(0)\,\}=\{0\}$, and as in Example$3.3$ one checks that $\{0\}$ is an H–ideal.

Lemma 3.5. Let $\{a_i\}$ and $\{b_i\}$, $i\in I$ be families of real numbers in $[0,1]$. Let $\alpha=inf_i a_i,\ \beta=inf_i b_i$. Then $inf_i \min(a_i,b_i)\geq\min(\alpha,\beta)$

Proof. For each i we have $a_i\geq\alpha$ and $b_i\geq\beta$, hence

$min(a_i,b_i)\geq min(\alpha,\beta), i\in I$. Therefore, $inf_{i\in I}\min(a_i,b_i)\geq\min(\alpha,\beta)$.

Theorem 3.6. Infimum of P–fuzzy ideals is a P–fuzzy ideal.

Let $\{f_i,\ i\in I\}$ be a family of P–fuzzy ideals on X.

Define $f: X\to[0,1]$ by $f(x)=inf_{i\in I} f_i(x)$. Then f is a P–fuzzy ideal.

Proof. f is a P–fuzzy ideal.

For each i and each x, $f_i(0) \geq f_i(x)$. Taking inf over i:

$f(0) = inf_i\, f_i(0) \geq inf_i\, f_i(x) = f(x)$. Also, for each i and each x, y, z: $f_i(x) \geq min(\, f_i((x * z) * (y * z)), f_i(y)\,)$. Taking inf over i: $f(x) = inf_i\, f_i(x) \geq inf_i\, min(\, f_i((x * z) * (y * z)), f_i(y)\,)$.

By Lemma3.5 again with $\left(a_i = f_i\big((x * z) * (y * z)\big), b_i = f_i(y)\right)$:

$inf_i\, min(\, f_i((x * z) * (y * z)), f_i(y)\,) \geq min(\, inf_i\, f_i((x * z) * (y * z)), inf_i\, f_i(y)\,)$.

Thus: $f(x) \geq min(\, f((x * z) * (y * z)), f(y)\,)$, which is (P−F2). Therefore, f is a P−fuzzy ideal.

Theorem 3.7. Infimum of H−fuzzy ideals is an H−fuzzy ideal.

Let $\{f_i,\ i \in I\}$ be a family of H−fuzzy ideals on X.

Define $f : X \rightarrow [0,1]$ by $f(x) = inf_{i \in I}\, f_i(x)$. Then f is a H−fuzzy ideal.

Proof. The same argument as in Theorem 3.6 shows that f is $(\mathrm{H} - \mathrm{F1})$ holds.

For the $(\mathrm{H} - \mathrm{F1})$ condition, for each i and for all x, y, z:

$f_i(x * z) \geq min(\, f_i(x * (y * z)), f_i(y)\,)$. Taking inf over i:

$f(x * z) = inf_i\, f_i(x * z) \geq inf_i\, min(\, f_i(x * (y * z)), f_i(y)\,)$.

Using Lemma 3.5 with $\left(a_i = f_i\big(x * (y * z)\big),\ b_i = f_i(y)\right)$:

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$inf_i\, min(\, f_i(x * (y * z)), f_i(y)\,) \geq\ min(inf_i\, f_i(x * (y * z)), iinf_i\, f_i(y)\,)$. So, $f(x * z)\ \geq\ min(\, f(x * (y * z)), f(y)\,)$. Hence f is an H−fuzzy ideal.

4. Conclusion

In this work we studied structural aspects of P-fuzzy and H-fuzzy ideals in BCI-algebras. For a P-fuzzy (resp. H-fuzzy) ideal f on a BCI-algebra X, we showed that the level subset A_f is a P-ideal (resp. H-ideal) of X, which links fuzzy ideals with the corresponding crisp ideals defined on the same algebra. We illustrated these results by constructing explicit examples of P-fuzzy and H-fuzzy ideals on a finite BCI-algebra.

Using a lemma concerning infima of real families, we then proved that the class of all P-fuzzy ideals (resp. H-fuzzy ideals) of a BCI-algebra is closed under arbitrary pointwise infima. Hence these classes admit natural complete meet-semilattice structures inside the lattice of all fuzzy subsets of the algebra. These basic properties form a starting point for a more detailed investigation of the lattice of P-fuzzy and H-fuzzy ideals, and suggest several directions for future work, such as studying distributivity and modularity conditions and comparing these fuzzy ideals with other generalized ideals that have been introduced in BCI-algebras.

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